42nd International Symposium on Forecasting

Revisiting Forecasting for Emergency Department Staffing

Bahman Rostami-Tabar[1], **Shixuan Wang**[2], and Stephen Disney[3]

[1]Cardiff Business School, Cardiff University

[2]Department of Economics, University of Reading

[3]University of Exeter Business School

July 11th 2022, Oxford, UK

What do we do?

We develop a model to generate categorical forecasts that corresponds to the staff-to-patients ratio requirements.

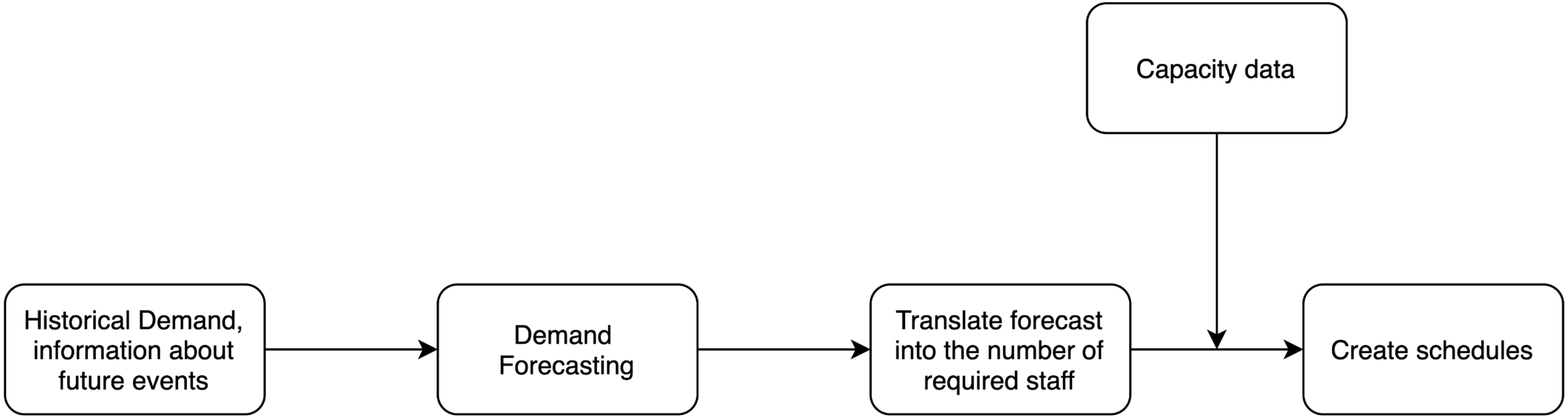
We compare the forecast accuracy of the proposed model using against benchmarks such as Naïve, Exponential Smoothing, ARIMA, TBATS & Random Forest.

We provide recommendations for staffing based on newsvendor model.

Outline

- Staffing & Forecasting in Emergency Departments
- Data
- Forecasting model
- Forecast evaluation
- Beyond forecast accuracy - newsvendor problem

Workflow in Emergency Departments



Original Data

One hospital in Cardiff

Every patients' arrival time

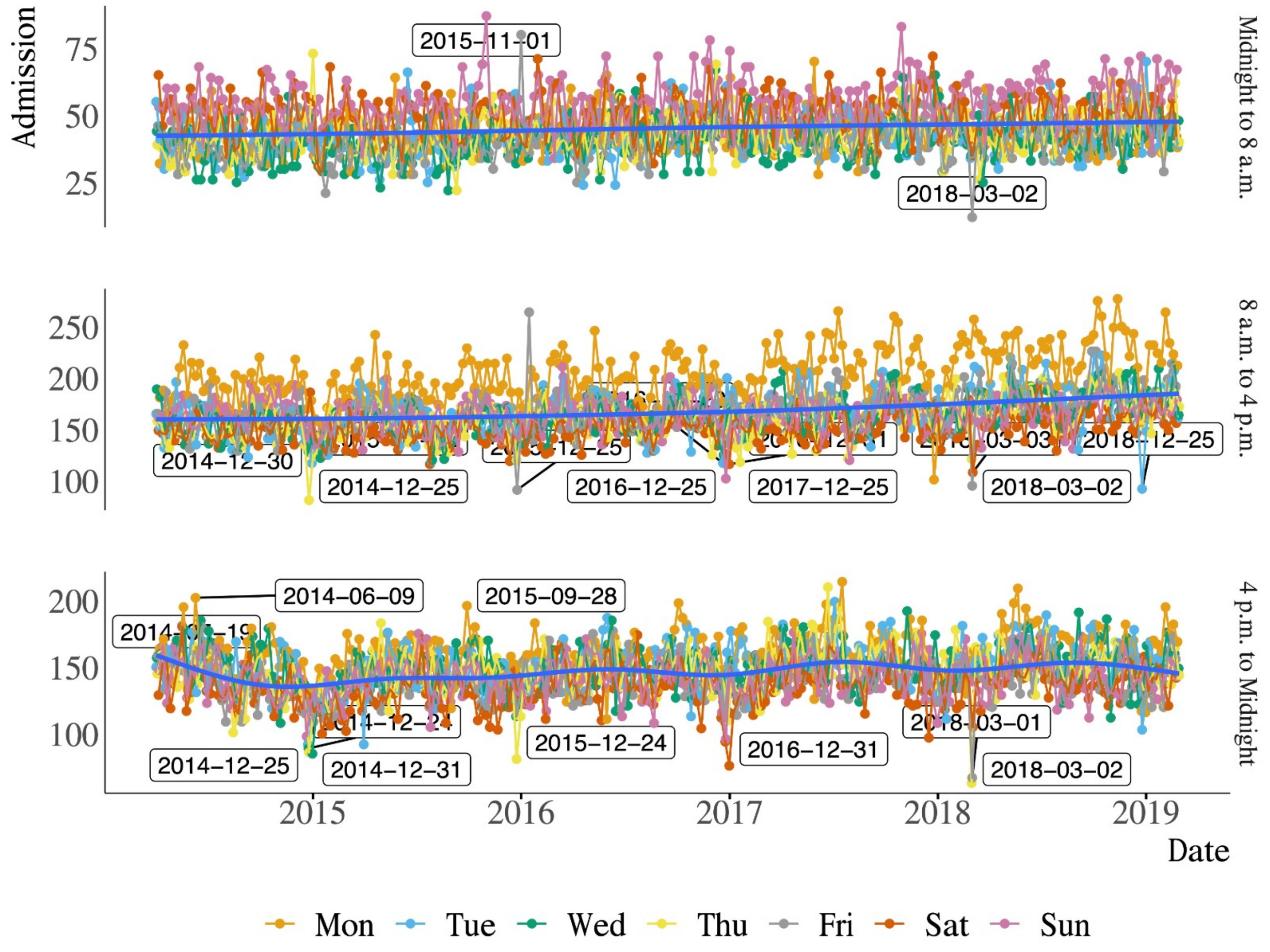
Period:
April 2014 – February 2019

We aggregate the **number of patients** in three shifts of each day.

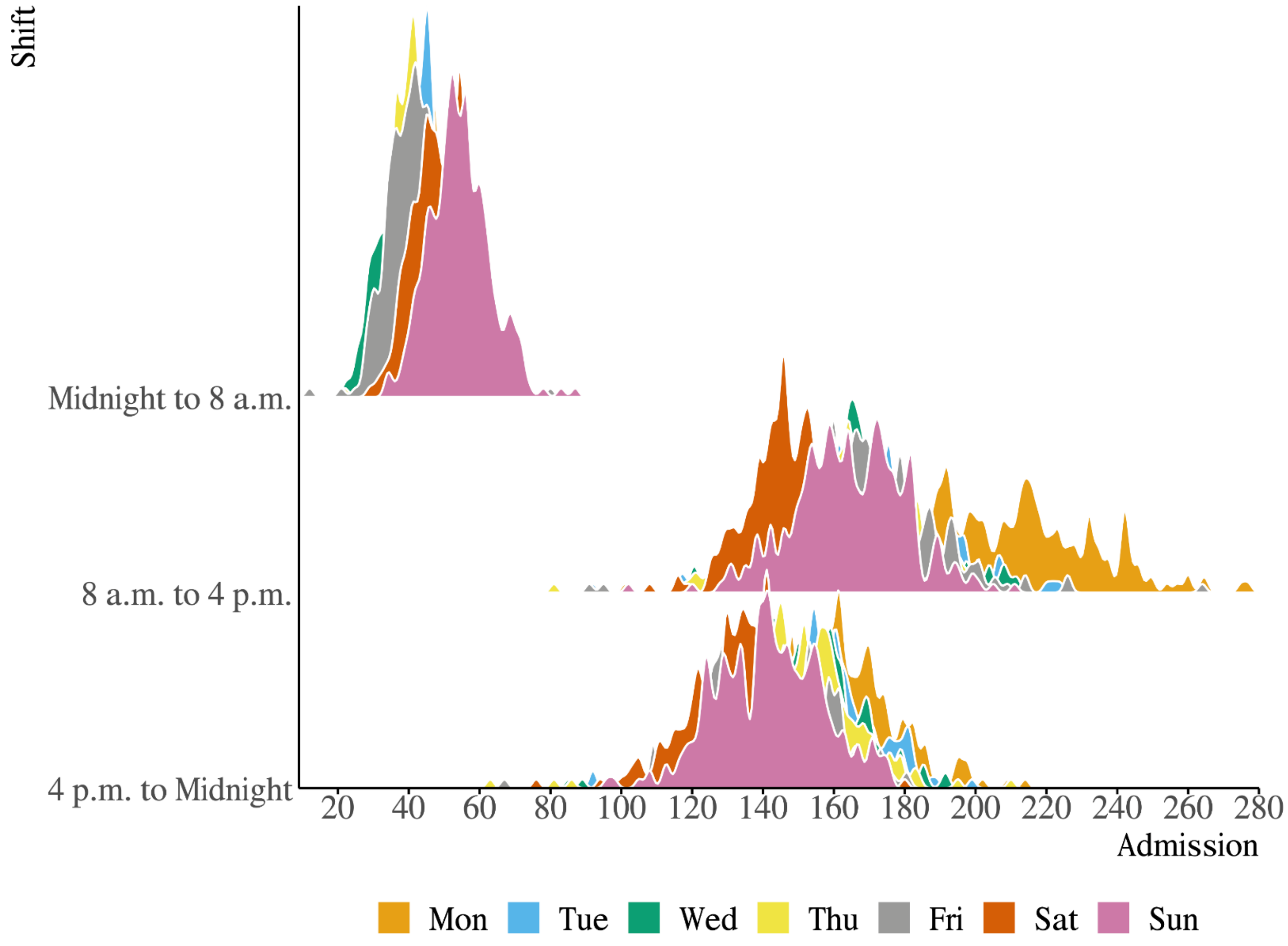
- Shift 1: 0:00 – 8:00
- Shift 2: 8:00 – 16:00
- Shift 3: 16:00 – 24:00

	Shift 1	*Shift 2*	*Shift 3*	All
Mean	45.08	168.06	146.26	119.80
SD	9.08	24.78	18.07	56.68
Skewness	0.46	0.82	-0.07	-0.33
Kurtosis	3.55	4.44	3.68	1.75
JB test	84.62	351.91	35.50	437.96
p-value	0.00	0.00	0.00	0.00
Min	12	81	63	12
25%-Q	39	151.75	134	51
Median	45	165	146	141
75%-Q	51	180	158	162
Max	87	277	214	277

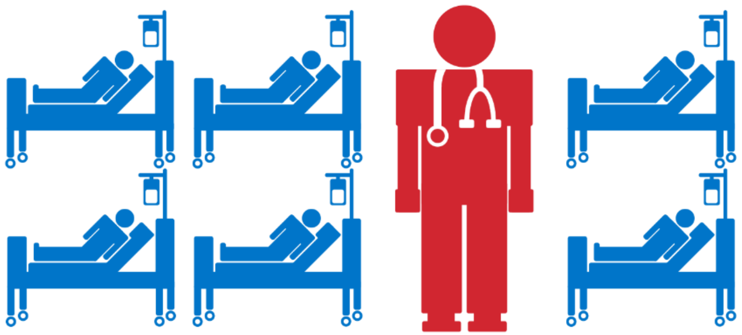
Time Series Plot of Each Shift

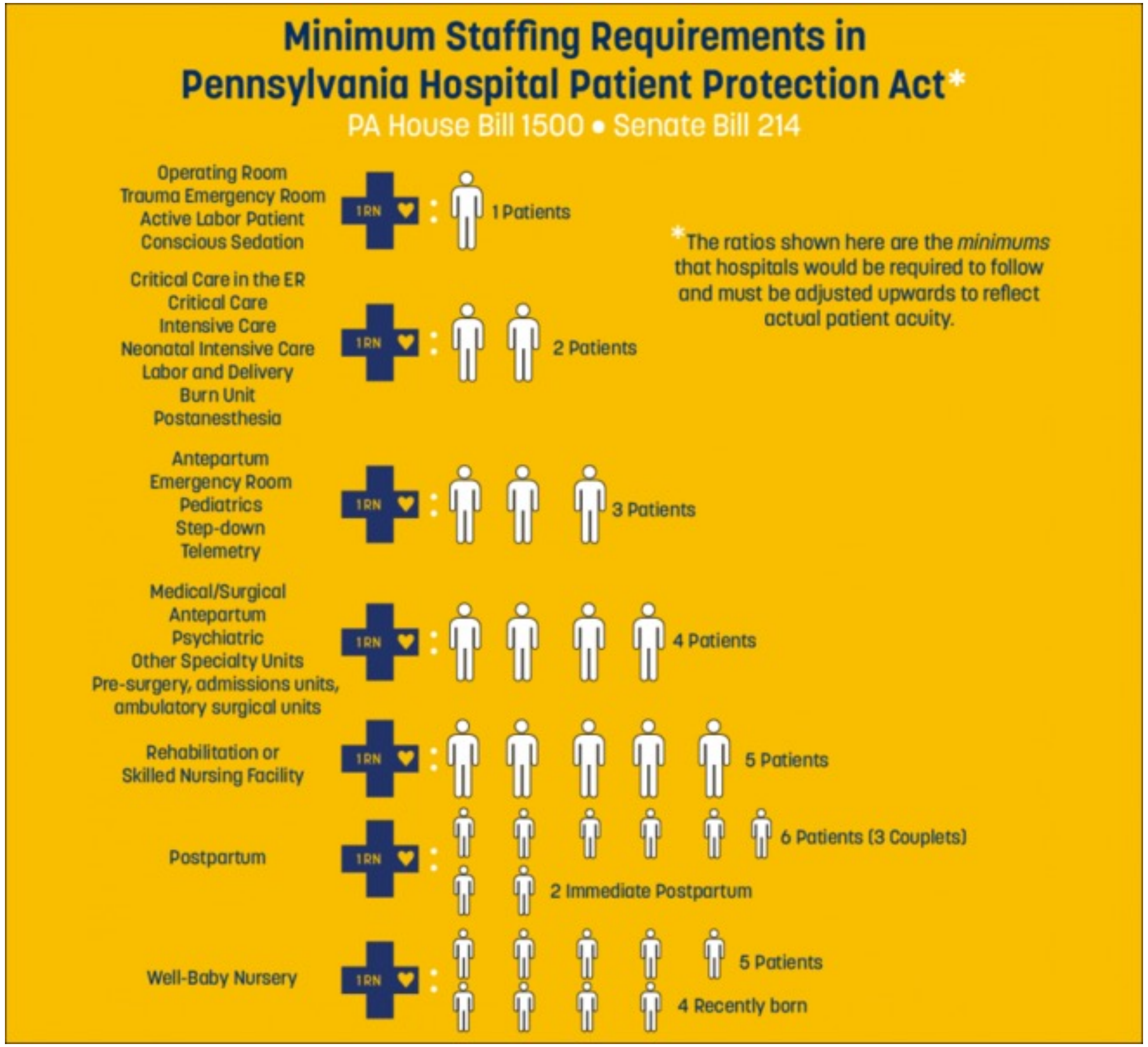


Distribution of Three Shifts in Each Day-of-Week



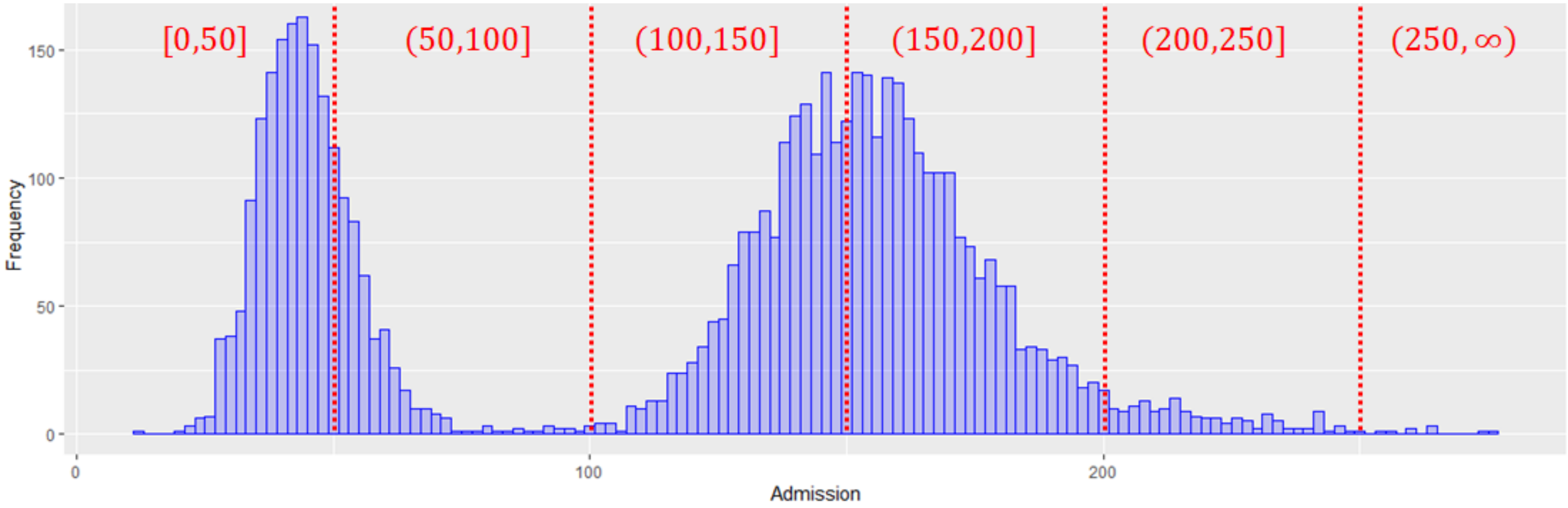
Staff-to-Patient Ratios





Why Categorical Forecast, rather than Point Forecast?

- The ultimate purpose of this study which is to support the planning of the emergency department.
- The planning is not going to change within a specific range.
- We convert the numerical number of patients into the following fix categories:



Methodology: Ordinal Regression

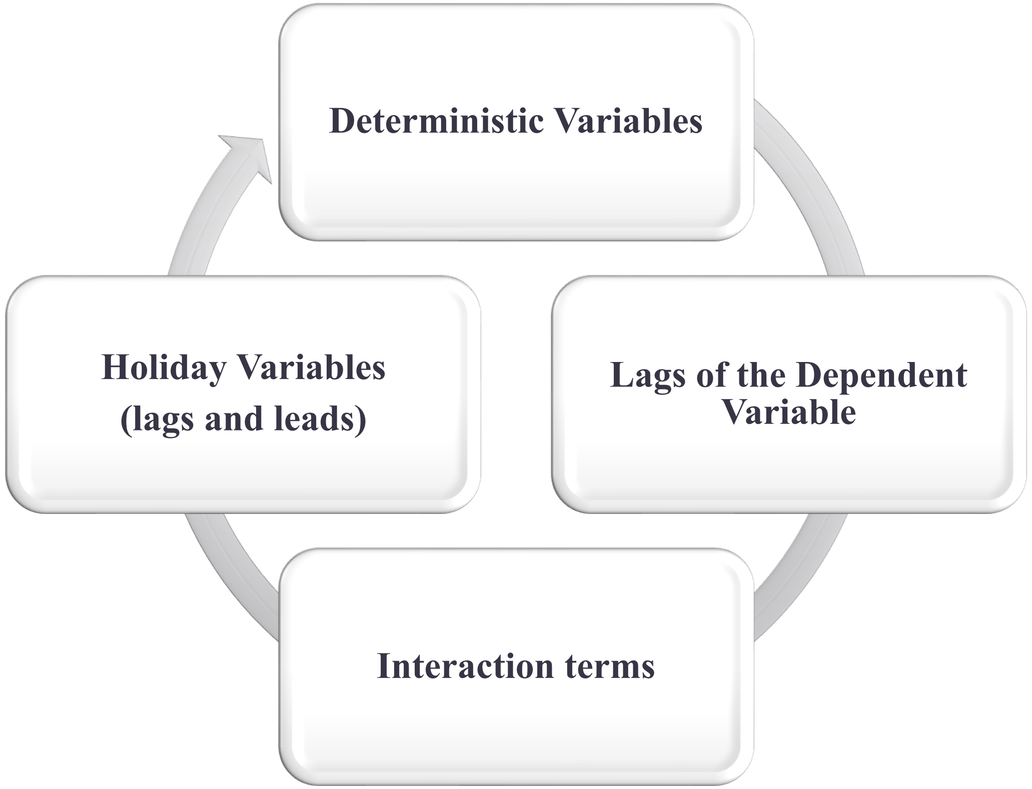
The ordinal regression is formulated in the following equation

$$\ln\left(\frac{P(y_t \le k)}{1-P(y_t \le k)}\right) = \alpha_{0,k} + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \ldots + \beta_p x_{p,t} + \varepsilon_t$$

where y_t is the dependent variable that belongs to one of the K categories, i.e. $y_i \in \{1,2,\ldots,K\}$; $x_{1,t},\ldots x_{p,t}$ are predictor variables; $\beta_{0,k},\ldots,\beta_p$ are unknown parameters to be estimated; ε_i is the error term.

- LHS: the **log of the odds** that an event occurs.
- RHS: the linear combination of the explanatory variables.

Choice of predictor variables: Four groups



Deterministic: shifts, time trend, month-of-year, and day-of-week.

Holidays: public holidays, school holidays, celebration day, rugby matches, and their lags and leads.

Lags: lags of the dependent variable.

Interactions: shifts and calendar variables.

Predictor Variables

- $shift_t$: Shift 1, Shift 2, or Shift 3.
- $trend_t$: a linear trend.
- $month_t$: calendar month.
- day_t: day of week.
- $isPubHol_t$: whether it is a public holiday
- $isSchHol_t$: whether it is a school holiday
- $isCelDay_t$: whether it is a celebration day
- $HolName_t$: specific holiday (e.g. Good Friday, Easter Monday, etc).
- $isRugCar_t$: Six Nations playing in Cardiff
- $isRugOut_t$: Six Nations playing in other countries
- $h_{i,t-3}$: the one day lag of the holiday variables
- $h_{i,t+3}$: the one day lead of the holiday variables
- $y_{t-3:t-90}$: the lagged dependent variables
- $shift_t \times h_{i,t}, shift_t \times h_{i,t-3}, shift_t \times h_{i,t+3}$: interaction terms of $shift_t$ and holiday variables (with their lag and lead).

Ordinal logit model with holiday, lag, and lead effects (OLM-HLL)

$$\ln\left(\frac{\mathbb{P}(y_t \leqslant k)}{1-\mathbb{P}(y_t \leqslant k)}\right) = \alpha_{0,k} + \beta_1 shift_t + \beta_2 trend_t + \beta_3 month_t + \beta_4 day_t$$
$$+ \sum_{i=1}^{6} \gamma_i h_{i,t} + \sum_{i=1}^{6} \phi_i h_{i,t-3} + \sum_{i=1}^{6} \theta_i h_{i,t+3} + \sum_{i=3}^{90} \rho_i y_{t-i}$$
$$+ \sum_{i=1}^{6} \omega_i (shift_t \times h_{i,t}) + \sum_{i=1}^{6} \kappa_i (shift_t \times h_{i,t-3})$$
$$+ \sum_{i=1}^{6} \delta_i (shift_t \times h_{i,t+3}) + \varepsilon_t$$

Methodology: LASSO *(Tibshirani, 1996)*

- LASSO: least absolute shrinkage and selection operator

- The objective function of LASSO has the penalized (negative) log-likelihood:

$$\mathrm{M}_{\lambda}(\boldsymbol{\beta}) = -\frac{1}{N}\sum_{i=1}^{N} \ln P(Y = y_i | \boldsymbol{X_i}, \boldsymbol{\beta}) + \lambda \sum_{j=1}^{P} |\beta_j|$$

where λ is the tuning parameter which controls the level of penalization.

- The parameters are estimated by minimize the objective function

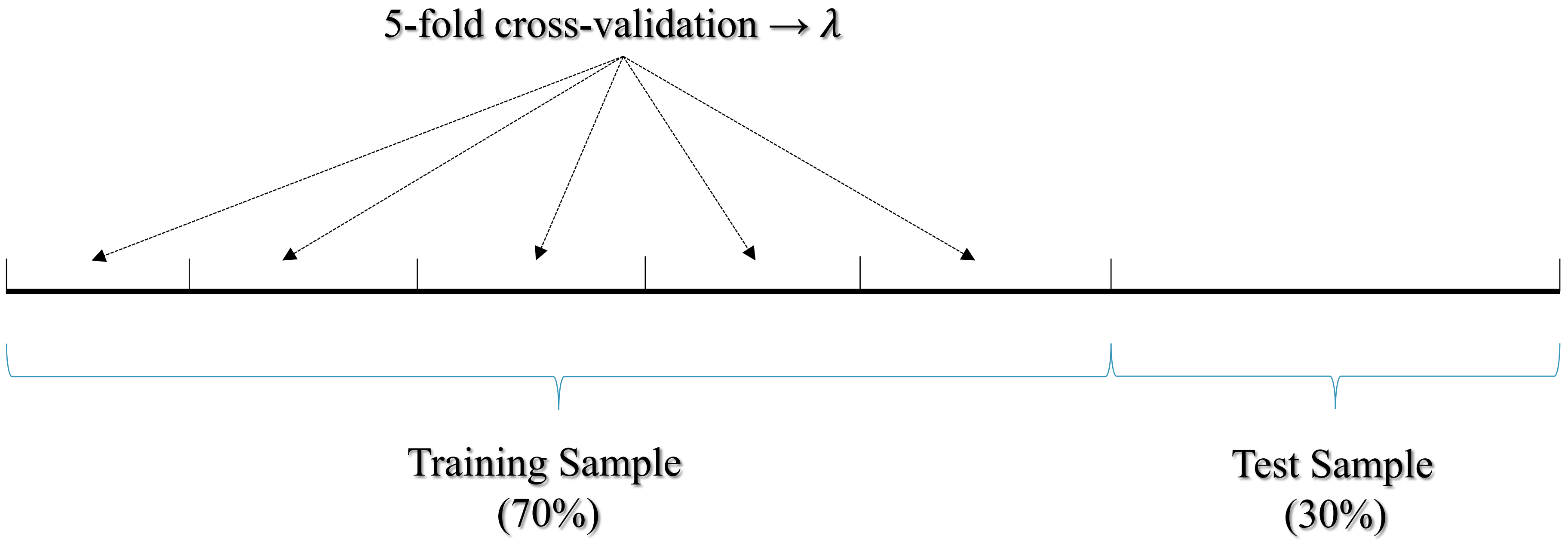
$$\widehat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \mathrm{M}_{\lambda}(\boldsymbol{\beta})$$

- Due to the penalization, many parameters tend to be zero, which automatically performs variable selections.
- Results in a sparse statistical model which only has a small number of nonzero parameters.
- It represents a classic case of “less is more”: a ‘sparse’ model can be much easier to estimate and interpret than a ‘dense’ model.

Benchmarks

- Seasonal naïve model
- ETS
- ARIMA
- TBATS
- Random Forecasts

Forecasting Experiment Design



Probabilistic Forecast Output

Shift	(0,50]	(50,100]	(100,150]	(150,200]	(200,250]	(250,300]
1	0.00%	5.40%	72.10%	22.40%	0.10%	0.00%
2	73.40%	26.50%	0.10%	0.00%	0.00%	0.00%
3	0.00%	1.30%	54.30%	43.80%	0.60%	0.00%
4	0.00%	1.00%	50.80%	47.30%	0.80%	0.00%
5	42.20%	56.30%	1.50%	0.00%	0.00%	0.00%
6	0.00%	0.00%	8.60%	75.30%	16.00%	0.00%
7	0.00%	0.40%	36.70%	60.80%	2.10%	0.00%
8	49.50%	49.50%	0.90%	0.00%	0.00%	0.00%
9	0.00%	0.00%	0.00%	6.60%	73.70%	19.70%

Forecasting Evaluation

- *Brier Score (Brier, 1950) –* ***the lower, the better!***

$$BS = \frac{1}{K}\sum_{k=1}^{K}\left(\hat{f}_k - o_k\right)^2$$

- *Ranked Probability Score (Epstein, 1969) -* ***the lower, the better!***

$$RPS = \frac{1}{K-1}\sum_{k=1}^{K}\left(\sum_{s=1}^{k}\hat{f}_k - \sum_{s=1}^{k} o_k\right)^2$$

where K is the number of possible categories, $\hat{f}_k$ and o_k are the forecasted probability and actual outcome for category j of observation i.

Forecasting Performance (interval = 50)

Brier Score

	LASSO	Naive	ARIMA	ETS	TBATS	RF
h=3	**0.196**	0.275	0.228	0.233	0.208	0.210
h=21	**0.197**	0.294	0.233	0.235	0.222	0.211
h=84	**0.203**	0.338	0.241	0.243	0.274	0.218
h=126	**0.200**	0.349	0.244	0.247	0.311	0.223

Ranked Probability Score

	LASSO	Naive	ARIMA	ETS	TBATS	RF
h=3	**0.040**	0.061	0.048	0.049	0.042	0.045
h=21	**0.041**	0.070	0.049	0.049	0.046	0.045
h=84	**0.042**	0.093	0.051	0.052	0.065	0.047
h=126	**0.041**	0.101	0.052	0.053	0.080	0.048

Note: Lower BS and RPS are better

Forecasting Performance – Other Interval Setting

	Brier Score							Ranked Probability Score					
	LASSO	Naïve	ARIMA	ETS	TBATS	RF		LASSO	Naïve	ARIMA	ETS	TBATS	RF
Interval of 40													
h=3	0.210	0.283	0.228	0.233	0.222	0.228		0.038	0.057	0.042	0.043	0.039	0.043
h=21	0.210	0.310	0.233	0.235	0.234	0.228		0.038	0.065	0.043	0.044	0.043	0.043
h=84	0.212	0.362	0.242	0.242	0.293	0.233		0.038	0.088	0.045	0.046	0.063	0.044
h=126	0.214	0.373	0.245	0.244	0.329	0.238		0.039	0.096	0.046	0.047	0.078	0.045
Interval of 30													
h=3	0.224	0.309	0.252	0.241	0.230	0.230		0.030	0.047	0.034	0.034	0.030	0.033
h=21	0.224	0.352	0.258	0.243	0.242	0.231		0.030	0.057	0.035	0.034	0.033	0.033
h=84	0.223	0.393	0.268	0.249	0.294	0.235		0.029	0.079	0.037	0.036	0.051	0.034
h=126	0.225	0.403	0.271	0.251	0.332	0.238		0.029	0.087	0.038	0.037	0.065	0.035
Interval of 20													
h=3	0.310	0.372	0.329	0.324	0.315	0.512		0.035	0.050	0.037	0.037	0.034	0.080
h=21	0.308	0.399	0.333	0.326	0.325	0.515		0.035	0.060	0.038	0.038	0.037	0.083
h=84	0.311	0.428	0.340	0.330	0.369	0.523		0.034	0.083	0.040	0.040	0.056	0.090
h=126	0.308	0.434	0.343	0.332	0.398	0.527		0.034	0.090	0.041	0.041	0.070	0.092

Insights for Operational Management for Emergency Departments

- Underage in emergency departments: shortage of staffs can cause long waiting, delayed treatment, etc.
- Coverage in emergency departments: surplus of staffs is a waste of recourses.
- The optimal staffing can be determined by Newsvendor model:
    - A decision variable (Q)
    - Uncertain demand (D)
    - Unit underage cost (c_u)
    - Unit overage cost (c_o)

Optimal Staffing based on Newsvendor Model

The optimal staff quantity for the emergency department can be found by finding the smallest Q such that

$$F(Q) = \sum_{D=0}^{Q} p(D) \geq \frac{c_u}{c_u + c_o}$$

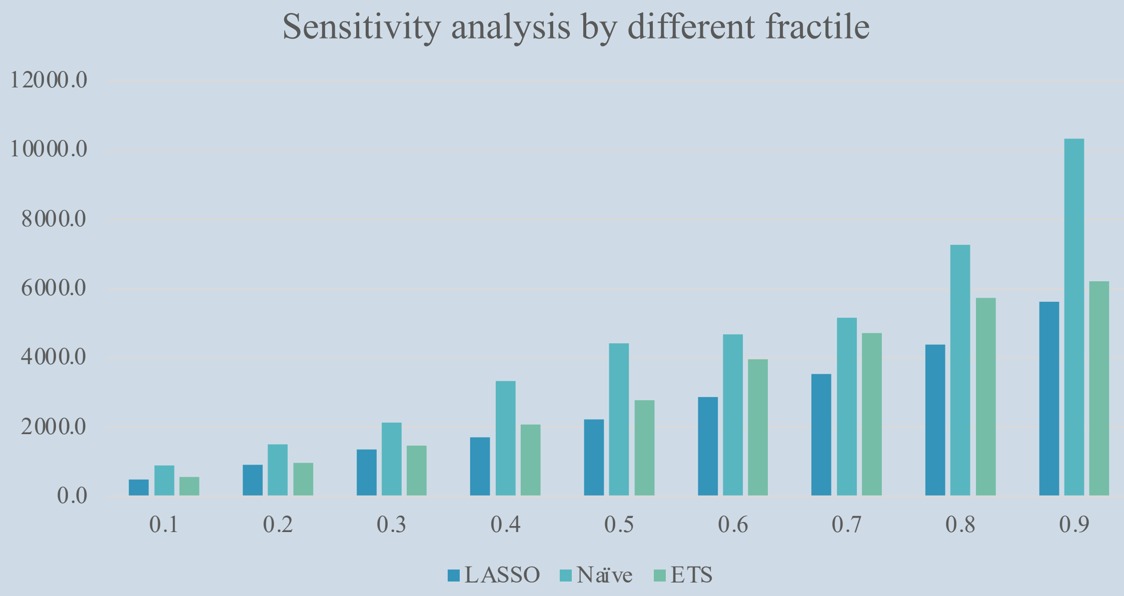
where the quantity

$$R = \frac{c_u}{c_u + c_o}$$

is the critical fractile.

Cost Calculation (h=21, 1-week)

Fractile	c_u	c_o	LASSO	Naïve	ETS
0.1	0.56	5	488.3	889.4	561.1
0.2	1.25	5	908.8	1511.3	965.0
0.3	2.14	5	1350.7	2137.1	1476.4
0.4	3.33	5	1705.0	3331.7	2075.0
0.5	5.00	5	2220.0	4415.0	2785.0
0.6	7.50	5	2870.0	4682.5	3962.5
0.7	11.67	5	3545.0	5158.3	4723.3
0.8	20.00	5	4390.0	7265.0	5735.0
0.9	45.00	5	5615.0	10335.0	6205.0



Summary

- We develop a categorical forecasting framework for emergency departments.
- We consider a large number of predictor variables, coupled with suitable variable selection techniques, LASSO.
- The forecasting performance of our model is superior to five benchmarks.
- The staffing decision based on newsvendor model and LASSO can potentially help to save costs for emergency departments.

CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

PANALPINA
Logistics & Manufacturing Research Centre at Cardiff

BT

EPSRC
Engineering and Physical Sciences Research Council

F4SG Research Grant - $5,000

The grant can be used for:
• Research capability building in developing countries.
• Collaborative research activity on forecasting for social good leading to publications in top journals such as *International Journal of Forecasting*.
• Promote the use of forecasting for social good in practice.

Lead applicant must be a researcher based on a low or lower-middle income country located in one of the following regions:
• Sub-Saharan Africa
• Latin America & Caribbean
• South Asia
• Middle East & North Africa

- **Important dates**

Application submission deadline: 31 Oct 2022
For any queries, please contact Dr. Rostami-Tabar (Rostami-tabarb@cardiff.ac.uk) or Dr. Shixuan Wang (shixuan.wang@reading.ac.uk).

- More information can be found via: https://forecasters.org/programs/research-awards/forecasting-for-social-good-research-grant/